Finding our Bearings on Slider Bearings:

Designing a Slider Bearing

Final Report

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Slider Bearing Material: Aluminum 6061-T6

Lubricant: Canola Oil

Estimated Distance bearing will travel: 34.7 inches

Problem Statement

The goal of this design is to create a slider bearing that, when struck by a pendulum at a 45 degree angle, will travel from rest to a predicted distance on 1 foot wide by 8 foot long aluminum track whose inclination angle is 0 degrees. The materials at the point of impact of the pendulum are a rubber ball. The design of the slider bearing will be proven using mathematical concepts and theory in tribology.

Slider bearing designs can be modeled after but are not limited to be a Rayleigh Step Bearing, Parallel Surface Bearing and a Step incline bearing. The lubricant choice is left up to the students but it must be nontoxic. The lubricant will have an effect on how far the slider bearing will go and some of the design properties of the bearing.

The slider bearing must have a mass greater than or equal to 500 grams and a width and length of less than or equal to 10 cm. The minimum machinable step size is 0.001 inches according to Joe Gomez, the Technical Staff Assistant who will be machining the bearing. Three different materials are available. These materials are Nylon 6,6; Cold Rolled Steel 1018; and Aluminum 6061-T6. The densities of all these materials are different and will change how the slider bearing performs.

Nomenclature

C_1, C_2

Constants of Integration

e

Coefficient of Restitution (Dimensionless)

g, g_x, g_y, g_z

Gravitational constant (9.81 m/s^2)

h_i, h_o

Height (m)

KE_0, KE_1

Kinetic Energy (J)

L

Length (m)

m_A, m_B

Mass (kg)

P

Pressure (Pa)

PE_0, PE_1

Potential Energy (J)

q_{in}, q_{out}

Flow Rate (m^3/s)

t

Time (s)

U

Velocity (m/s)

V , V_{A1}, V_{A2}, V_{B1}, V_{B2}, V_x, V_y, V_z

Velocity (m/s)

Greek Symbols

η

Viscosity (Pa*s)

ρ

Density (kg/m^3)

Design Solution

The material chosen for this design is Aluminum 6061-T6 due to its easy machinability and low surface roughness. It has a density of 2.7 g/cm^3 . The lubricant of choice is canola oil, which has a viscosity of 57 mPa*s.[1] Canola oil was chosen due to its high viscosity, general availability and non-toxic nature.

The following equation was used to determine the total mass of the bearing, taking into consideration the overall dimensions of the block of aluminum as well as the material removed to form the step:

Initial Design

$$Mass = \rho V$$

$$Mass = \rho * Length * Width * Height$$

$$Mass = 2700\frac{kg}{m^3} * 0.1m * 0.03m * 0.0635m$$

Mass=0.514kg

$$Mass\ of\ Step = 2700\frac{kg}{m^3}0.000312m * 0.0201m * 0.1m$$

$$Mass\ of\ Step = 0.002kg$$

$$\boxed{\text{Total Mass=0.5124kg=512g}}$$

The Pendulum

First, the conservation of energy equation is used to help determine the energy at the point of impact between the pendulum ball and the slider bearing. Since m and g are given and h can be calculated by eq. **(2)**, the equation can be solved for v.

$$PE_0 + KE_0 = PE_1 + KE_1 \ \mathbf{(1)}$$

$$mgh + 0 = 0 + \frac{1}{2}mv^2$$

$$h = (L - Lcos(\theta)) \ \ \mathbf{(2)}$$

Plugging eq. **(2)** into eq. **(1)** yields:

$$g * (L - Lcos(\theta)) = \frac{1}{2}mv^2 \ \ \mathbf{(3)}$$

Plugging in accepted value of g (9.81 $\frac{m}{s^2}$) and the given values (L=0.36 m, θ=45°):

$$9.81\frac{m}{s^2} * (0.36m - 0.36\cos(45)) = \frac{1}{2}v^2$$

Solving for v provides the velocity of the pendulum ball immediately before impact:

$$\boxed{\text{V=1.438}\frac{m}{s}}$$

Coefficient of Restitution

In order to find the velocities of the ball, and more importantly, the slider bearing after impact, we must utilize the Coefficient of Restitution equation, eq. **(4)**

$$e = \frac{V_{B2} - V_{A2}}{V_{A1} - V_{B1}}$$

Plugging in the V value calculated from eq. **(3)** as V_{A1} and the estimated value of e (for rubber e=0.7)[2] and solving for $V_{B2} - V_{A2}$:

$$0.7 = \frac{V_{B2} - V_{A2}}{1.43 \frac{m}{s}}$$

Solving for $V_{B2} - V_{A2}$:

$$\boxed{1.002 \frac{m}{s} = V_{B2} - V_{A2}}$$

Momentum

In order to solve for the values of $V_{B2} - V_{A2}$, the momentum equation, eq. **(5)**, must be implemented using the given values for m_A and m_B and the determined value V_{A1} :

$$m_A V_{A1} + m_B V_{B1} = m_A V_{A2} + m_B V_{B2}$$

$$(0.215kg)\left(1.43 \frac{m}{s}\right) = (0.215kg)V_{A2} + (0.5kg)V_{B2}$$

$$.30745N = (0.215kg)V_{A2} + (0.5kg)V_{B2}$$

Plugging these values into an rref equation:

$$\text{rref}\begin{vmatrix} (0.215kg)V_{A2} & (0.5kg)V_{B2} & 0.30745N \\ -1V_{A2} & 1V_{B2} & 1.002N \end{vmatrix}$$

$$\boxed{\begin{array}{c} V_{A2}=-0.27m/s \\ V_{B2}=0.732m/s \end{array}}$$

V_{B2} is the initial velocity of the bearing after being struck by the pendulum.

Order of Magnitude Analysis

For this portion of the analysis, a long-bearing (B>>L ~ Y>>X) is assumed. This is possible because the movement and forces present in the X-Z direction are many orders of magnitude greater than the movement and forces present in the Y-Z direction. This will allow many terms will cancel out in the Navier-Stokes equations. Using a thin film approximation H<<L:

$$\frac{1}{H} \gg \frac{1}{L}$$

$$\frac{\partial}{\partial z} \gg \frac{\partial}{\partial x}$$

In this case, V_z can be eliminated because $V_x \gg V_z$

$$V_x \sim U\left(\frac{L}{L}\right) \dots . \sim 1$$

$$V_z \sim U\left(\frac{H}{L}\right) \dots . \ll 1$$

Continuity

$$\cancel{\frac{\partial P}{\partial t}} + \cancel{\frac{\partial \rho V_x}{\partial x}} + \frac{\partial \rho V_y}{\partial y} + \frac{\partial \rho V_z}{\partial z} = 0$$

$$\frac{\partial V_x}{\partial x} = -\frac{\partial V_z}{\partial z}$$

Reduction of Navier-Stokes

$$\text{X-Direction}: \rho\left(\overset{\boxed{6}}{\cancel{\frac{\partial V_x}{\partial t}}} + \overset{\boxed{4a}}{\cancel{V_x \frac{\partial V_x}{\partial x}}} + \overset{\boxed{2}}{\cancel{V_y \frac{\partial V_x}{\partial y}}} + \overset{\boxed{4b}}{\cancel{V_z \frac{\partial V_x}{\partial z}}}\right) = -\frac{\partial P}{\partial x} + \eta\left(\overset{\boxed{3}}{\cancel{\frac{\partial^2 V_x}{\partial x^2}}} + \overset{\boxed{2}}{\cancel{\frac{\partial^2 V_x}{\partial y^2}}} + \frac{\partial^2 V_x}{\partial z^2}\right) + \overset{\boxed{5}}{\cancel{\rho g_x}}$$

$$\text{Y:Direction}: \cancel{\rho\left(\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z}\right) = -\frac{\partial P}{\partial y} + \eta\left(\frac{\partial^2 V_y}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2} + \frac{\partial^2 V_y}{\partial z^2}\right)} + \rho g_y \quad \boxed{2}$$

$$\text{Z Direction}: \rho\left(\underset{\boxed{6}}{\cancel{\frac{\partial V_x}{\partial t}}} + \underset{\boxed{4a}}{\cancel{V_x \frac{\partial V_z}{\partial x}}} + \underset{\boxed{2}}{\cancel{V_y \frac{\partial V_z}{\partial y}}} + \underset{\boxed{4b}}{\cancel{V_z \frac{\partial V_z}{\partial z}}}\right) = \underset{\boxed{7}}{\cancel{-\frac{\partial P}{\partial z}}} + \eta\left(\underset{\boxed{4a}}{\cancel{\frac{\partial^2 V_z}{\partial x^2}}} + \underset{\boxed{2}}{\cancel{\frac{\partial^2 V_z}{\partial y^2}}} + \underset{\boxed{3}}{\cancel{\frac{\partial^2 V_z}{\partial z^2}}}\right) + \cancel{\rho g_z} \quad \boxed{5}$$

1. Incompressible
2. 2-D Approximation
3. Continuity
4. Order of Magnitude Analysis
    a) H<<L
    b) Vx>>Vz
5. No Body Forces
6. Steady State
7. Pressure Gradient =0

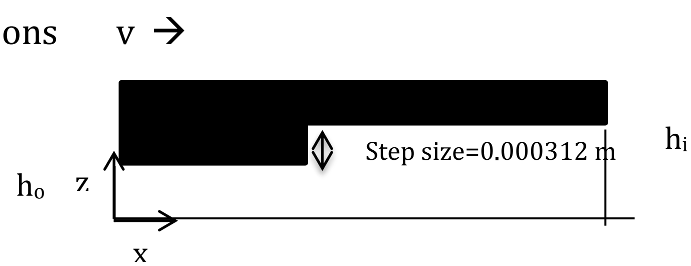
Reduced Navier Stokes

$$\text{X-Direction :}\quad \frac{1}{\eta}\frac{\partial P}{\partial x} = \frac{\partial^2 V_x}{\partial z^2}$$

This is a diagram of a Rayleigh step bearing. In order to find h_i, the flow rate must be determined at the inlet and the outlet of the bearing and set equal to one another. Regardless of the film thickness, the flow rate always remains the same, which causes an increase in pressure at the step. This occurs because the same flow rate is trying to travel through a smaller volume, exerting more pressure. To find the flow rate q, the velocity profiles at the inlet and outlet must be determined. The initial velocity profile equation must be integrated twice, which gives **eq. (6)** :

Determining Step Size

Boundary Conditions



Z=0; V_x =0

Z=h_i; V_x =U (inlet)

Z=h_o; V_x=U (outlet)

Integrate to find the velocity profiles at the inlet and the outlet:

$$\frac{\partial^2 V_x}{\partial z^2} = \frac{1}{\eta}\frac{\partial P}{\partial x}$$

$$\frac{\partial V_x}{\partial z} = \frac{1}{\eta}\frac{\partial P}{\partial x} z + C_1$$

$$V_{x=}\frac{1}{\eta}\frac{\partial P}{\partial x}\frac{z^2}{2} + C_1 z + C_2 \qquad \textbf{(6)}$$

Since the double integration yields two constants of integration, the boundary conditions must be used. First, the boundary condition Z=0; U=0 is used:

$$0 = \frac{1}{\eta}\frac{\partial P}{\partial x}\frac{0^2}{2} + C_1 0 + C_2$$

C_2=0

Now that C2 has been determined, the inlet boundary condition, Z=h_i; U=U, is used:

$$U = \frac{1}{\eta}\frac{\partial P}{\partial x}\frac{{h_i}^2}{2} + C_1 h_i$$

$$C_1 = \frac{U}{h_i} - \frac{1}{\eta}\frac{\partial P}{\partial x}\frac{h_i}{2}$$

Plugging C1 and C2 back into eq. **(6),** the **inlet velocity profile** is given:

$$V_{x=}\frac{1}{2\eta}\frac{\partial P}{\partial x}z^2 + (\frac{U}{h_i} - \frac{1}{2\eta}\frac{\partial P}{\partial x}h_i)z$$

$$\boxed{V_{x=}\frac{1}{2\eta}\frac{\partial P}{\partial x}(z^2 - zh_i) + \frac{U}{h_i}z}$$

Now to determine the velocity profile of the outlet, the boundary condition Z=0; U=0 is plugged into eq. **(6):**

$$0 = \frac{1}{\eta}\frac{\partial P}{\partial x}\frac{0^2}{2} + C_1 0 + C_2$$

C_2=0

Now that C2 has been found, utilize the outlet boundary condition Z=h_o; U=U to find C1:

$$U = \frac{1}{\eta}\frac{\partial P}{\partial x}\frac{{h_o}^2}{2} + C_1 h_o$$

$$C_1 = \frac{U}{h_o} - \frac{1}{\eta}\frac{\partial P}{\partial x}\frac{h_o}{2}$$

Plugging C1 and C2 back into eq. **(6)** gives the **outlet velocity profile**:

$$V_{x=}\frac{1}{2\eta}\frac{\partial P}{\partial x}z^2 + (\frac{U}{h_o} - \frac{1}{2\eta}\frac{\partial P}{\partial x}h_o)z$$

$$\boxed{V_{x=}\frac{1}{2\eta}\frac{\partial P}{\partial x}(z^2 - zh_o) + \frac{U}{h_o}z}$$

Now that the velocity profiles have been determined for the inlet and the outlet, the flow rates for the inlet and outlet can be determined by integrating the velocity profile from 0 to h, starting with the inlet flow rate:

$$q_{in} = \int_0^{h_i} V_x dz$$

$$q_{in} = \int_0^{h_i} \frac{1}{2\eta}\frac{\partial P}{\partial x}(z^2 - zh_i) + \frac{U}{h_i} z\, dz$$

$$q_{in} = \frac{1}{6\eta}\frac{\partial P}{\partial x} z^3 - \frac{1}{4\eta}\frac{\partial P}{\partial x}\ z^2 h_i + \frac{U}{h_i}\frac{z^2}{2}\ \Big|_0^{h_i}$$

$$q_{in} = \frac{1}{6\eta}\frac{\partial P}{\partial x} {h_i}^3 - \frac{1}{4\eta}\frac{\partial P}{\partial x}\ {h_i}^2 h_i + \frac{U}{2h_i} {h_i}^2$$

Simplifying the output of the integral:

$$\boxed{q_{in} = -\frac{{h_i}^3}{12\eta}\frac{\partial P}{\partial x} + \frac{U}{2} h_i}$$

Now that the inlet flow rate has been determined, the outlet can be determined using the same method:

$$q_{out} = \int_0^{h_o} V_x dz$$

$$q_{out} = \int_0^{h_o} \frac{1}{2\eta}\frac{\partial P}{\partial x}(z^2 - zh_o) + \frac{U}{h_o} z\, dz$$

$$q_{out} = \frac{1}{6\eta}\frac{\partial P}{\partial x} z^3 - \frac{1}{4\eta}\frac{\partial P}{\partial x}\ z^2 h_o + \frac{U}{h_i}\frac{z^2}{2}\ \Big|_0^{h_o}$$

$$q_{out} = \frac{1}{6\eta}\frac{\partial P}{\partial x} {h_o}^3 - \frac{1}{4\eta}\frac{\partial P}{\partial x}\ {h_o}^2 h_o + \frac{U}{2h_o} {h_o}^2$$

Simplifying the output of the integral

$$\boxed{q_{out} = -\frac{{h_o}^3}{12\eta}\frac{\partial P}{\partial x} + \frac{U}{2} h_o}$$

Simplifying Reynolds Equation

$$\overset{\boxed{3}}{\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial P}{\partial x}\right)} + \overset{\boxed{2}}{\cancel{\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\eta}\frac{\partial P}{\partial y}\right)}} = 6(U_1+U_2)\overset{\boxed{4}}{\cancel{\frac{\partial(h\rho)}{\partial x}}} + 12\overset{\boxed{1}}{\cancel{\frac{\partial(h\rho)}{\partial t}}}$$

Assumptions

1. Assume there is no squeezing
2. Assume there is no side leakage
3. Assume an incompressible Fluid
4. Assume there is no change in film thickness in the x direction

Boundary Conditions

X=0; $P=P_{atm}=0$

X=L; $P=P_{atm}=0$

X=n ; $P=P_{max}$

X=L-n ; $P=P_{max}$

After reducing Reynolds Equations, the equation becomes:

$$\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial x}\right) = 0$$

Integrating in terms of x two times yields:

$$\frac{\partial P}{\partial x} = C_1$$

$$P = C_1 x + C_2$$

Now, utilizing the boundary condition X=0; $P=P_{atm}=0$, C_2 can be determined:

$$0 = C_1 0 + C_2$$

$$C_2 = 0$$

Which leaves:

$$P = \frac{\partial P}{\partial x} x$$

As stated earlier, the pressure at the step needs to be the same using X=L-n ; $P=P_{max}$ and X=n ; $P=P_{max}$. From above equation:

$$\left(\frac{\partial p}{\partial x}\right)_i = \frac{p_{max}}{n}$$

$$\left(\frac{\partial p}{\partial x}\right)_o = -\frac{p_{max}}{L-n}$$

Utilizing these two equations and equating them to one another (q_i=q_o)

$$-\frac{{h_i}^3}{12\eta}\frac{\partial P}{\partial x} + \frac{U}{2}h_i = \frac{{h_o}^3}{12\eta}\frac{\partial P}{\partial x} + \frac{U}{2}h_o$$

$$-\frac{{h_i}^3}{12\eta}\frac{p_{max}}{n} + \frac{U}{2}h_i = \frac{{h_o}^3}{12\eta}\frac{p_{max}}{L-n} + \frac{U}{2}h_o$$

Now, arranging the equation to solve for P_{max}:

$$P_{max} = \frac{6\eta U(h_i - h_o)}{\frac{{h_i}^3}{n} + \frac{{h_o}^3}{(L-n)}}$$

Plugging $H = \frac{h_i}{h_o}$ *and* $\beta = \frac{n}{L-n}$ into the P_{max} equation:

$$\frac{P_{max}}{6\eta U} = \frac{(H*h_o - h_o)}{\frac{H*{h_o}^3}{\beta(L-n)} + \frac{{h_o}^3}{(L-n)}}$$

$$\boxed{P_{max} = \frac{6\eta U(L-n)}{{h_o}^2}\frac{\beta(H-1)}{H^3+\beta}}$$

Using the information that B/L should be greater than 4 to classify it as no side leakage and utilizing the Matlab code (*attachment #1*), the best ratio that can be maintained with length and weight restrictions is 3.33. Shows there will be some side leakage. The selected dimensions, as stated earlier are L=3cm, B=10cm and H=6.35cm. Now the load per unit width must be determined to prove the accepted value of β(2.588):

$$\frac{W}{B} = \frac{1}{2}Pmax(n + (L-n))$$

Sub in Pmax and simplify

$$\frac{W}{B} = \frac{\eta U L^2}{{h_o}^2}\frac{3\beta(H-1)}{(1+\beta)(H^3+\beta)}$$

Using ideal conditions and the width is infinite it can be shown that β=2.55 and H=1.87 by taking the derivative of the dimensionless load term

$$W *= (\frac{3\beta(H-1)}{(1+\beta)(H^3+\beta)})$$

Derivative of W * with respect to H:

$$\frac{3(H-1)}{(1+\beta)(H^3+\beta)} - \frac{9\beta(H-1)H^2}{(1+\beta)(H^3+\beta)^2}$$

Solving for β:

$$\beta = 2H^3 - 3H^2$$

The derivative with respect to P:

$$\frac{3(H-1)}{(1+\beta)(H^3+\beta)} - \frac{3\beta(H-1)}{(1+\beta)^2(H^3+\beta)} - \frac{3\beta(H-1)}{(1+\beta)(H^3+\beta)^2}$$

Plug in $\beta$and solve for H:

$$\boxed{H=1.866}$$

So $\beta = 2.55$ (2.588 in notes and reference books)

Plugging these into W*=0.20626

Since this slider bearing design is not infinite H value is slightly lower than 1.81~1.77; $\frac{n}{L}$ is slightly lower than 0.69 ~0.67.[3]

In order to determine the β ratio, the following equation is used:

$$\frac{0.67}{1} = \frac{n}{3cm}$$

n=2.01cm

L-n=0.99cm

$$\beta = 2.03$$

$$W *= \left(\frac{3(2.03)(1.79-1)}{(1+2.03)(1.79^3+2.03)}\right) = .20448$$

Plugging this back into the load per unit length equation the following is obtained:

$$\boxed{\frac{W}{B} = \frac{\eta U L^2}{{h_o}^2} W^*}$$

Plugging in the determined and accepted values, the height of the output can be determined:

$$W=0.514kg*9.81\frac{m}{s^2}$$

$$\eta=0.057Pa*s$$

$$U=0.752m/s$$

$$L=0.03m$$

$$B=0.1m$$

$$\frac{0.514(\text{kg})*9.81(\frac{m}{s^2})}{0.1(m)}=\frac{0.057(Pa*s)*0.752(\frac{m}{s})*0.03(m)^2}{{h_o}^2}*0.2043$$

Rearranging the equation to solve for h_o yields:

$$\boxed{h_o = 0.000395m\ or\ 0.397mm}$$

Using the relation $1.79=\frac{h_i}{.000397(m)}$:

$$\boxed{h_i = 0.000703m}$$

To determine the step size, h*o* must be subtracted from h_i:

$$Step\ Size = 0.000703m - 0.000397m = 0.000314m$$

$$\boxed{\text{Step size=0.000312 m}}$$

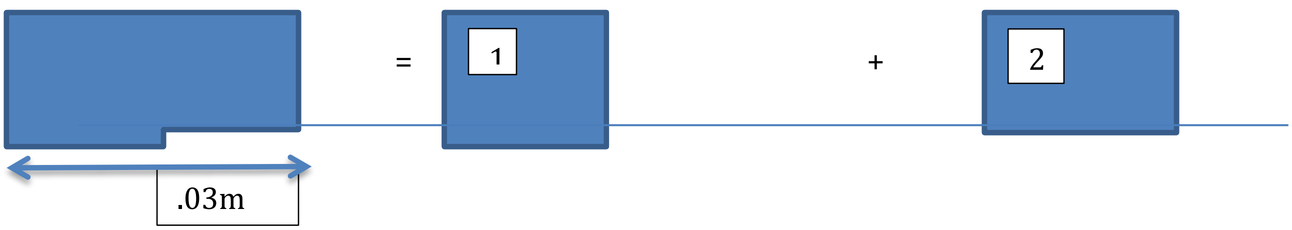
Now with the step size calculated, a SolidWorks model (attachment two) can be created using both the chosen dimensions and the calculated step size.

Developing the Final Design

Design Solutions

In the formal report, a major flaw in the data appeared. Without the center of gravity of the bearing above the step, the bearing would tip forwards after contact with the pendulum. This would occur because the pressure of the lubricant ahead of the step would not be sufficient enough to overcome both the normal force of the bearing and the moment created by the collision. This would be corrected in the final design by placing the center of gravity over the step, which is the point of greatest pressure in the bearing. The following page will illustrate the transition between the old design and the new including all justifications.

In order to determine the center of mass of the original bearing, it must be separated into two pieces at the step, as the diagram on the following page explains:



Segment	Area	X bar	Y bar	XbarA	YbarA
1	6.2865e-4m²	0.00495m	0.03175m	3.112e-6m³	1.996e-5m³
2	0.00127m²	0.01995m	0.03175m	2.534e-5m³	4.0325e-5m³
	0.00190			2.845e-5m³	6.028e-5m³

$$Center\ of\ Mass\ x = \frac{\Sigma xbarA}{Area}$$

$$x\ Center\ of Mass = \frac{2.845e-5m^3}{0.00190m} = 0.0149m$$

$$Center\ of\ Mass\ y = \frac{\Sigma ybarA}{Area}$$

$$y\ Center\ of\ Mass = \frac{6.0285e-5m^3}{0.00190m} = 0.03175m$$

Mass Balance

Mass of Segment 1

$$0.0099\text{m} * 0.0635\text{m} * 0.1\text{m} * 2700\,\frac{kg}{m^3}\text{=0.1697kg}$$

Mass of Segment 2

$$0.0201\text{m} * (0.0635\text{m} - 0.000312\text{m}) * 0.1\text{m} * 2700\,\frac{kg}{m^3}\text{=0.3429kg}$$

The resulting difference of the masses is 0.1732kg

Now, the masses must be set equal to one another to determine the new heights.

New Mass of Segment 1:

$$0.0099m * Height(m) * 0.1m * 2700\,\frac{kg}{m^3} = 0.2563kg$$

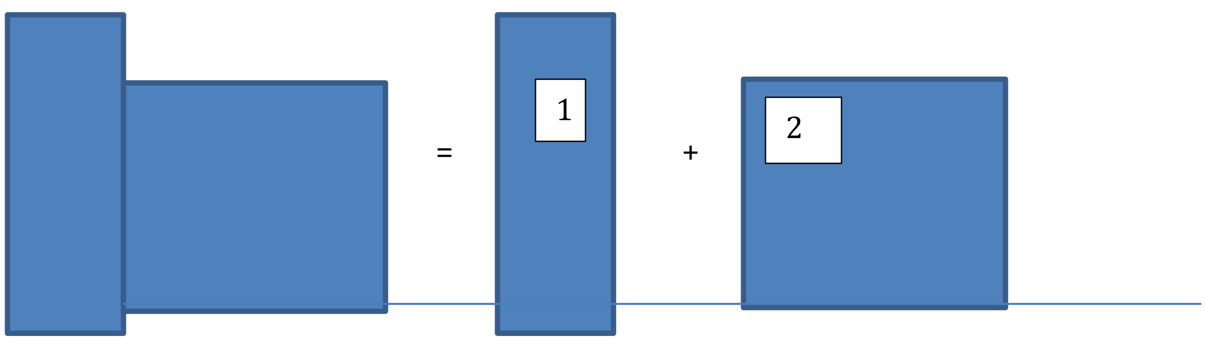
New Mass of Segment 2:

$$0.0201m * Height(m) * 0.1m * 2700\,\frac{kg}{m^3} = 0.2563kg$$

New Height of Segment 1: 0.0959m

New Height of Segment 2: 0.04722m

Now the center of gravity must be checked again:



Segment	Area	X bar	Y bar	XbarA	YbarA
1	9.494e-4m^2	0.00495m	0.04795m	4.7e-6e-5m^3	4.552e-5m^3
2	9.428e-4m^2	0.01995m	0.02361m	1.881e-5m^3	2.226e-5m^3
	0.00189			6.778e-5m^3	.0358e-5m^3

$$x\ Center\ of\ Mass = \frac{2.351e-5m^3}{0.00189m} = 0.0124m$$

$$y\ Center\ of\ Mass = \frac{6.778e-5m^3}{0.00189m} = 0.0358m$$

Right away this design shows that it is not good. First the center of mass is still not over step so the bearing will fall over. Second the center of mass in the y direction has gone up. It is critically important that the bearing is hit as close to the center of mass as possible so the slider will go straight. If the slider is hit below the center of mass it will buck backwards and if it is hit above, it will tilt forward. This is the design flaw with the above bearing. It is possible to make the center of mass in the x direction over the step but it will even further increase the y center of mass so it is not worth exploring changing the dimensions of this bearing anymore.

Instead the length of the step will have to change and become smaller. Using the matlab code and eventually changing the parameters so it looks at ratios between 2 and 1 a B/L ratio of 1 was chosen. Although this violates what was stated earlier that a B/L must >4 to neglect side leakage this change must be made so the slider can stand up and not fall over. The length that was determined was a length of 55mm a width of 55mm and a height of 63.5mm.f

Using the parameters proposed by Rohde of ratios, the step size, film thickness and step length have to be recalculated. Now since the B/L =1, The film thickness ratio H~1.69, and the Step Ratio $\frac{n}{L}$~0.55.[3]

The mass of this bearing without the step is

$$.055m * .055m * .0635m * 2700\ kg/m\text{^}3\ = 0.5186kg$$

This is a high mass so if .5cm was taken off the length it will decrease the mass. The length was taken off the length rather than the width so the B/L ratio will stay above 1.

The new mass

$$.055m * .0545m * .0635m * 2700\ kg/m\text{^}3\ = 0.514kg$$

In order to determine the β ratio, the following equation is used:

$$\frac{0.55}{1} = \frac{n}{5.45cm}$$

n=3.025cm

L-n=2.425cm

$$\beta = 1.24$$

$$W^* = \left(\frac{3(1.25)(1.69-1)}{(1+1.25)(1.69^3+1.25)}\right) = 0.188$$

Plugging this back into the load per unit area equation the following is obtained:

$$\boxed{\frac{W}{B} = \frac{\eta U L^2}{{h_o}^2} W^*}$$

Plugging in the determined and accepted values, the height of the output can be determined:

W=0.518kg*9.81$\frac{m}{s^2}$

η=0.057Pa*s

U=0.732m/s

L=0.055m

B=0.055m

$$\frac{0.518(\text{kg}) * 9.81(\frac{m}{s^2})}{0.055(m)} = \frac{0.057(Pa * s) * 0.732(\frac{m}{s}) * 0.055(m)^2}{{h_o}^2} * 0.188$$

Rearranging the equation to solve for h_o yields:

$$\boxed{h_o = 0.000507m\ or.507mm}$$

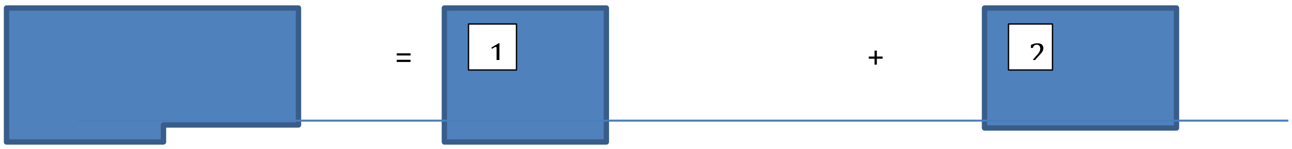
Using the relation 1.79=$\frac{h_i}{.000397(m)}$:

$$\boxed{h_i = 0.000857m}$$

To determine the step size, ho must be subtracted from h_i:

$$Step\ Size = 0.000857m - 0.000507m = 0.000350m$$

Step size=0.000350 m



Segment	Area	X bar	Y bar	XbarA	YbarA
1	0.00157e-4m^2	0.01237m	0.03175m	1.944e-5m^3	4.99e-5m^3
2	0.00191m^2	0.0398m	0.03175m	7.62e-5m^3	6.065-5m^3
	0.00348			9.56e-5m^3	1.105e-4m^3

$$Center\ of\ Mass\ x = \frac{\Sigma xbarA}{Area}$$

$$x\ Center\ of Mass = \frac{9.56e - 5m^3}{0.00348m} = 0.0275m$$

$$Center\ of\ Mass\ y = \frac{\Sigma ybarA}{Area}$$

$$y\ Center\ of\ Mass = \frac{1.105e - 4m^3}{0.00348m} = 0.03175m$$

Mass Balance

Mass of Segment 1

$$0.02425\text{m} * 0.055\text{m} * 0.0635\text{m} * 2700\,\frac{kg}{m^3} = 0.2286\text{kg}$$

Mass of Segment 2

$$0.03025\text{m} * (0.0635\text{m} - 0.00035\text{m}) * 0.055\text{m} * 2700\,\frac{kg}{m^3} = 0.2837\text{kg}$$

Difference in the masses is 0.0502kg

Make the masses equal each other

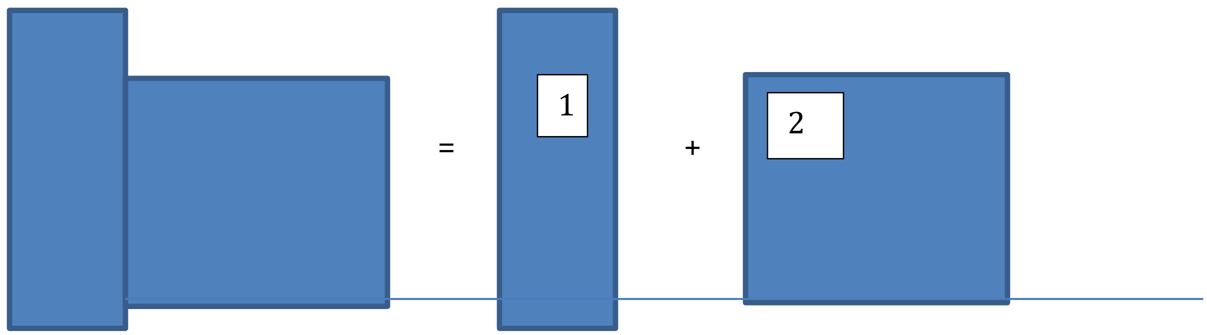
New Mass of Segment 1: $0.02425\text{m} * 0.055\text{m} * \text{Height m} * 2700\,\frac{kg}{m^3} = 0.2562\text{kg}$

New Mass of Segment 2: $0.03025\text{m} * \text{Height} * 0.055\text{m} * 2700\,\frac{kg}{m^3}$=0.2562kg

New Height of Segment 1: 0.071m

New Height of Segment 2: 0.057m

Check the center of Gravity again



Segment	Area	X bar	Y bar	XbarA	YbarA
1	0.00172m^2	0.0121m	0.0355m	2.087e-5m^3	6.1122e-5m^3
2	0.00173m^2	0.0393m	0.0285m	6.74e-5m^3	4.8839e-5m^3
	0.00189			8.83e-5m^3	1.099e-4m^3

$$x\ Center\ of\ Mass = \frac{8.83e-5m^3}{0.00189m} = 0.0257m$$

$$y\ Center\ of\ Mass = \frac{1.099e-5m^3}{0.00189m} = 0.0320m$$

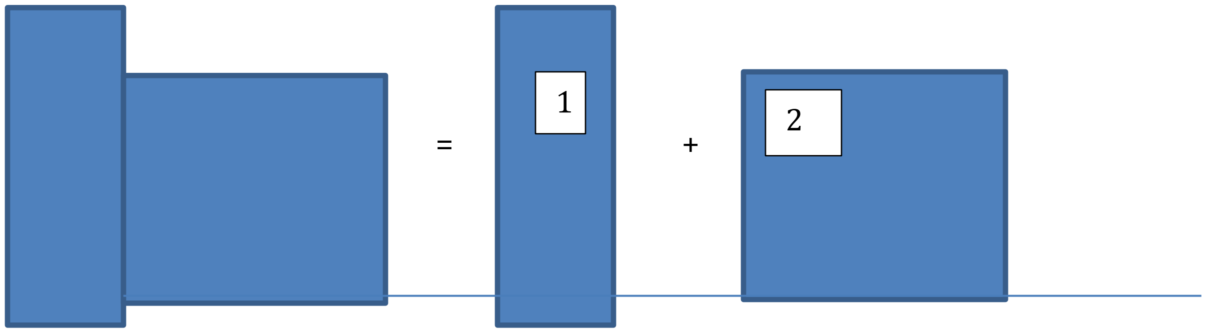
This shows that Segment 1 has to become a little taller and segment 2 a little shorter. The weight of the bearing can also be adjusted because it is still slightly high. To make the mass = to 500g take away 12.4g from the right side

$0.03025\text{m} * 0.055\text{m} * \text{Height m} * 2700\,\frac{kg}{m^3}$=0.0124kg

Height=0.00276m

New Height=0.057m-0.00276m=0.0542m

Check the center of Gravity again



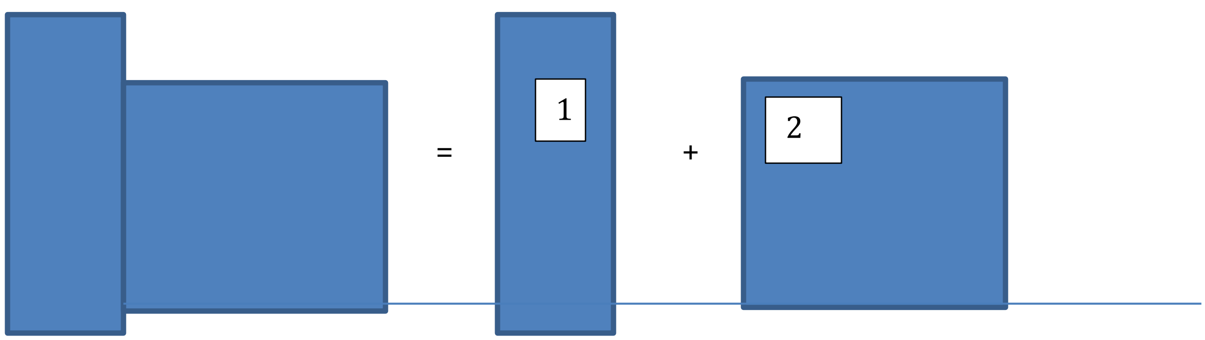
Segment	Area	X bar	Y bar	XbarA	YbarA
1	.00172m^2	0.0121m	0.0355m	2.087e-5m^3	6.1122e-5m^3
2	.00163m^2	0.0393m	0.0272m	6.45e-5m^3	4.4739e-5m^3
	0.00336			8.54e-5m^3	1.058e-4m^3

$$x\ Center\ of\ Mass = \frac{8.54e-5m^3}{0.00336m} = 0.0254m$$

$$y\ Center\ of\ Mass = \frac{1.058e-5m^3}{0.00336m} = 0.0315m$$

Adding some mass on the right side should be able to move the center of gravity up and to the left. This is how a final height for the left side of the slider bearing was calculated.

Center of Mass Calculations – Final Dimensions



Segment	Area	X bar	Y bar	XbarA	YbarA
1	.00174m^2	0.0121m	0.0359m	2.111-5m^3	6.25e-5m^3
2	.00163m^2	0.0393m	0.0272m	6.455e-5m^3	4.47e-5m^3
	0.00336			8.566e-5m^3	1.072e-4m^3

$$x\ Center\ of\ Mass = \frac{8.525e-5m^3}{0.00336m} = 0.0253m$$

$$y\ Center\ of\ Mass = \frac{1.066e - 5m^3}{0.00336m} = 0.0317m$$

As stated earlier, the reason for the drastic overhaul in the design is to place the center of gravity at the step in terms of the x direction and at the point of impact of the pendulum ball in the y direction. The center of gravity being on the step will allow for the point of maximum shear to fall directly on the point of maximum pressure (the step), and the shear will taper off in both x directions towards the front and back of the bearing in a linear way, much like the pressure distribution beneath the bearing. This correlation will provide pressure in appropriate amounts where it is needed the most, allowing for a very efficient overall design.

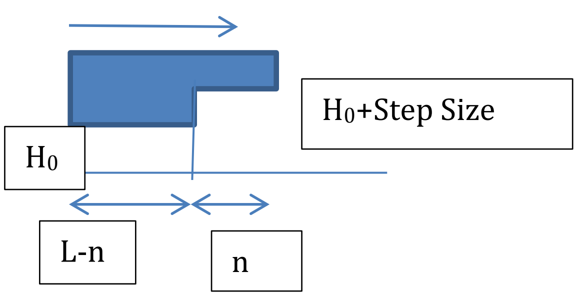
Determining the Distance Traveled

In order to determine the distance travelled, a very basic dynamic analysis must be done on the system consisting of the track, the lubrication and the bearing. The bearing will have an initial velocity, a normal velocity and a frictional force acting upon it. Since the initial velocity and the normal force have been determined earlier in the report, the frictional force is all that remains to be determined. This can be determined by multiplying the normal force by the frictional coefficient, which is calculated by taking the shear force acting on the bottom side of the bearing, f, and dividing it by the weight of the bearing, w_z.(Reference 4, pages 461-463)

Friction

$$f=\int_0^l \tau_{zx} = \int_0^l -\frac{h}{2}\frac{\partial p}{\partial x} - \frac{\eta U}{h}$$

The last equation on the previous page needs to be modified for the step bearing for the two sections of the bearing, separated by the step.[4]



$$f=\int_0^{l-n} \tau_{zx} dx + \int_{l-n}^{l} \tau_{zx} dx$$

Plugging in the velocity profile from the calculations the following is obtained:

$$\frac{f}{B} = \int_0^{l-n} -\frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta U}{h} dx + \int_{l-n}^{l} -\frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta U}{h} dx$$

$$f = \left(-\frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta U}{h}\right)\Big|_{0}^{l-n} + \left(-\frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta U}{h}\right)\Big|_{l-n}^{l}$$

$$\frac{f}{B} = \left(-\frac{h_0}{2}\frac{\partial p}{\partial x_o} + \frac{\eta U}{h_0}\right) * (l-n) + \left(-\frac{h_i}{2}\frac{\partial p}{\partial x_i} + \frac{\eta U}{h_i}\right)(l-(l-n))$$

$$\left(\frac{\partial p}{\partial x}\right)_i = \frac{P_{max}}{n}$$

$$\left(\frac{\partial p}{\partial x}\right)_o = -\frac{P_{max}}{l-n}$$

$$\frac{f}{B} = \left(\frac{h_0}{2}\frac{P_{max}}{l-n} + \frac{\eta U}{h_0}\right) * (l-n) + \left(-\frac{h_i}{2}\frac{P_{max}}{n} + \frac{\eta U}{h_i}\right)(l-(l-n))$$

Now, P_{max} must be solved for, as well as f to plug into the final coefficient of friction equation:

$$P_{max} = \frac{6\eta U(L-n)}{{h_o}^2}\frac{\beta(H-1)}{H^3+\beta}$$

$$= \frac{6 * 0.057\mathrm{Pa} * \mathrm{s} * .732\,\frac{\mathrm{m}}{\mathrm{s}} * (.02425\mathrm{m})}{.000507m^2} * \frac{1.25(1.69-1)}{1.69^3+1.25} = 3352.1Pa$$

$$\frac{W}{B} = \frac{\eta U L^2}{{h_o}^2}W^* = \frac{.057Pa * s * .732\,\frac{m}{s} * (0.0545m)^2}{(.000395m)^2} = 93.12\,\frac{N}{m}$$

Now that $\frac{W}{B}$ has been determined, $\frac{f}{B}$ must also be determined to solve for the coefficient of friction.

$$\frac{f}{B} = \left(\frac{0.000507m}{2}\frac{3352.1Pa}{0.02425m} + \frac{0.057Pa * s * 0.732\,\frac{m}{s}}{0.000507m}\right) * 0.02425m + \left(-\frac{0.000857m}{2}\frac{3352.1Pa}{0.03025m} + \frac{0.057Pa * s * 0.732\,\frac{m}{s}}{0.000857m}\right)(0.0545m - (0.02425m))$$

$$\frac{f}{B} = 2.879\frac{N}{m}$$

Now plugging into the μ equation:

$$\mu = \frac{\frac{f}{B}}{\frac{W}{B}} = \frac{2.879\frac{N}{m}}{93.12\frac{N}{m}} = 0.0309$$

Now that the coefficient of friction has been determined, the normal load and the initial velocity can assist in determining the total distance traveled by the bearing using the following equation:

$$0 = \frac{1}{2}mv^2 - (\mu * \mathrm{W}) * \mathrm{x}$$

$$x = \frac{\frac{1}{2}mv^2}{(\mu * \mathrm{W})}$$

$$x = \frac{v^2}{2\mu * g}$$

$$x = \frac{0.732^2}{2(.0309) * 9.81}$$

x =0.884m = 2.899 feet=34.7 inches

Conclusions

The redesign of the bearing was very beneficial. Having the center of gravity over the step in the x direction and in line with the pendulum impact in the y direction improves the efficiency of our bearing by keeping the front end of the bearing from tipping forwards. The new design allows the maximum amount of energy be transferred from pendulum to bearing with minimal losses. Losses in the old design would have consisted of a moment, either above or below the center of gravity that would have either tipped the bearing forward or backwards. The new design has effectively eliminated these losses.

The result of the calculations is a nearly cubic bearing (2.17”x2.15”x2.83”) with a step of 0.012 inches located 1.19 inches from the front of the bearing along the base. The calculated value of the coefficient of friction for an aluminum bearing with canola oil as lubricant is 0.0309. The total distance travelled by the bearing will be 2.899 feet.

Regarding Attachment One

This Matlab code works by looking at a slider with a height of 0.0635m. Now the code goes through all of the possible combinations of lengths and width from 1cm in increments of 1 mm. The line with the variable 'check' is how all of these choices are limited down to choices to applies to different slider bearings. For example only slider dimensions when the mass is over 500g are shown. If the slider dimensions satisfy this condition check is given the value of 1(true) if these conditions are not satisfied check is given a value of 0(false). At the end all of the checks that are true are shown. The number of conditions that can be checked are easily changed for different applications.

```
 %Nick DiFilippo and Andy Choquette
%Tribology Slider Bearing
%10/28/2010
%Aluminum 6061-T6

clc
clear all
CRS=2700; %kg/m^2
Mass=500e-3; %kg
Volume=Mass/CRS

W=.01:.001:.10;
L=.01:.001:.10;

   for i=1:91
       for j=1:91
           V=L(i)*W(j)*.0635; %Volume

            M=CRS*V;
            Ratio=W(j)/L(i);
            check=Volume<=.0635*W(j)*L(i) &M>500e-3 & M<520e-3 &W(j)>=L(i)&Ratio ==1 ;
                                                %Conditions to be checked, ensuring the mass is greater
                                                 %than 500g but less than 520 to narrow the results as
                                                 %well as making sure the length and width are at a 1:1 ratio
            if check==1
                fprintf('Mass= %f Length= %f; Width= %f; Ratio %f ', M, L(i), W(j), Ratio)
            end
        end

    end
```

References

1. "Viscosity." *The Physics Hypertextbook*. Web. 07 Dec. 2010. <http://physics.info/viscosity/>.

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